Full proposal for project *SingFlows*

*Sing*ular *Flows*: boundary layers, vortex filaments, wave-structure interaction.

Contents

Project Summary:

The objective of SingFlows is to develop mathematical and numerical tools for the analysis of three problems in fluid dynamics: the behaviour of anisotropic flows (boundary layers, shallow water flows), the dynamics of vortical structures, and the evolution of fixed or floating structures in water waves. Our will to unify these different problems is natural, because they share many mathematical features. The underlying keypoint is that they are described by singular solutions of Euler or Navier-Stokes equations. The word *singular* refers here:
- either to a lack of smoothness: it applies for instance to vortex filaments, which are Dirac masses along curves, or to the contact line between water and the floating structure,
- or to a singular dependence of the solution with respect to a parameter, typically the Reynolds number (like in boundary layers).
The connection between the two points of view is usually made by viscous regularization of the non-smooth structure, or conversely by taking the vanishing limit of the parameter.

More generally, the three problems considered in SingFlows involve flows with very small scales. A relevant description then requires the derivation of reduced models.

Beyond these common mathematical challenges, which are at the core of the project, the problems studied in SingFlows are intrinsically of an applied nature. They have concrete implications (for river flows, blood circulation, the wear of floating structures), and any quantitative understanding requires numerical simulations, as well as a knowledge of realistic settings. This is why the SingFlows project is based on an integrated team of 25 people with expertise in partial differential equations, numerical analysis, and in the computation and physics of fluid dynamics. This team has pre-existing connections, which will make the task implementation easier. This task implementation follows a careful schedule, with identification of short-, mid- and long-term objectives. It includes efforts towards the dissemination of our results, both to international specialists and general audience.

Several breakthrough results are expected:
- an improved description of friction laws in shallow water flows.

- the justification of the binormal flow approximation through a vanishing viscosity limit.
- the development of robust and efficient numerical codes for wave-structure interaction.

Summary table of persons involved in the project.
(Rk: the percentage of involvement is averaged over the four years.)

Partner 1 is IMJ-PRG (UMR 7586), Partner 2 is Université Grenoble-Alpes, Partner 3 is Université de Bordeaux.

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1 Context, positioning and objectives of the project

The project SingFlows aims at a better understanding of three topics in fluid dynamics:

i) The description of anisotropic flows, like boundary layers, shallow water or pipe flows.

ii) The description of vortex dynamics in slightly viscous fluids.

iii) The interaction between water waves and fixed or floating structures.

These topics originate from different contexts, but they have strong connections at the mathematical level. In all three settings, the flows can be described by Euler or Navier-Stokes type equations, and the associated solutions exhibit spatial singularities. For instance, the contact line between the water surface and a floating object is no more than Lipschitz. Vortex filaments in ideal fluids are modeled by Dirac masses along curves. In the Navier-Stokes evolution, such filaments are instantaneously regularised at positive times, but they still exhibit a singular dependence in the Reynolds number as it becomes large, similarly to what happens for boundary layers. From the numerical point of view, the common difficulty is that these singularities cannot be captured by direct computations, as they require too many grid points. It is then necessary to derive reduced models, that allow to retain the main features of the singular region, and/or its effect on the large scale flow.

The goal of the SingFlows project is to develop innovative mathematical methods, as well as efficient reduced models and codes, to improve the description of the singular flows involved in i)-ii)-iii).

1.1 Motivations

We first present the main motivations behind the SingFlows project.

Flows with small aspect ratio

Many flows are characterised by a small aspect ratio: their typical transverse scale is much smaller than their typical tangential scale. A classical example is geophysical flows, like rivers or oceanic currents. Another example is blood flow: indeed, the depth of veins and arteries varies between 2 mm and 2.5 cm, while their total length in the human body is 90000 km. We can also mention the boundary layers that develop in high Reynolds number flows near solid boundaries, or, in the context of oceanography, near coasts or within very long waves such as tsunamis. Understanding the dynamics of these anisotropic flows is crucial in many concrete issues. Many questions can be formulated regarding such issues, for example:

• How does an obstacle upstream of the flow influence the discharge downstream? This question is relevant to the construction of bridges in rivers, but also to airway obstructions in the body, in connection to sleep apnea.

• How does roughness at a solid surface affect energy dissipation in a fluid flow? This question is related to the role of bathymetry in rivers, or to the effect of arteries' stiffness on the blood flow. Conversely, computing the impact of friction forces on the erosion of solid surfaces is a major issue in mechanical engineering or geophysics.

To go beyond a simple qualitative description, the numerical simulation of anisotropic flows is necessary. The problem is that a direct simulation of the full Navier-Stokes equations is often too demanding computationally, especially if the anisotropic flow is coupled to a large scale one. On the other hand, one may hope to take advantage of the anisotropy through a dimensional analysis, which leads to rescaled Euler or Navier-Stokes equations with small parameters. These dimensionless systems open the road to the use of asymptotic expansions, so as to derive reduced models which are easier to handle numerically. Nevertheless, this idea faces *singular perturbation problems*: the terms of highest order in the models (typically the diffusion term in Navier-Stokes equations) often disappear in the asymptotics. This change of nature of the limit PDE system generates major mathematical challenges. First, finding the appropriate form of the asymptotic expansion can be difficult *per se*. Indeed, concentrations or oscillations generated by the singularities may create a complicated dependence on the small parameters. In particular, it is very difficult to go beyond the first term of the asymptotics, that is to include next order corrections. This difficulty appears for instance when computing roughness-induced effects on laminar flows [22], or in the classical periodic homogenization of elliptic systems [4]. This difficulty also arises in the modelling of friction terms, for instance in shallow water flows: indeed, the viscous terms disappear from leading order asymptotics. Deriving the appropriate corrective drag term is a major issue, and a major motivation for the SingFlows project.

Even once an appropriate expansion and some reduced model have been derived, difficulties remain, and it may happen that the approximate model does not properly describe the fluid dynamics. In the language of numerical analysis, the approximation can be consistent but not stable. Typically, in the derivation of simpler equations for anisotropic flows, one usually discards the tangential diffusion, which is formally negligible compared to the transverse one. This allows for high frequency instabilities, because the reduced equation does not contain (or at least contains less) smoothing mechanisms. Typical examples of such non-physical instabilities are provided by the Prandtl equation, for which various ill-posedness or blow-up results are known, see [36, 23] and references therein.

A natural problem is then to determine among different consistent systems which one to choose: ideally, it should retain the real instabilities and not suffer from artificial high frequency instabilities. Again, this very challenging problem is a main motivation for the SingFlows project.

Vortices

In incompressible flows, singularities always occur in connection with high concentration of vorticity. Tornadoes, for instance, are spectacular examples of columnar vortices, and for obvious reasons it is of great practical importance to understand how these structures arise, move over long distances, and eventually disappear. Whirlpools in oceans, or anticyclonic storms in geophysical flows (such as Jupiter's Great Red Spot) are other examples of persistent, nearly two-dimensional vortices. Vortex pairs can be observed in the wake of modern aircrafts, where they are created near the wingtips and the flaps. Such trailing vortices persist over long time scales, and this imposes severe restrictions on the frequency of take-offs and landings in busy airports. Vortex sheets occur in abrupt shear flows, and evolve into complicated structures through the Kelvin-Helmholtz instability. Finally, vortex rings are very common in turbulent flows, and can be observed for instance during volcanic eruptions, in physiological flows, or (in dramatic circumstances) around the main rotor of a helicopter. Understanding the dynamical behavior of vortices is then a problem of the utmost importance. At the mathematical level, the following questions arise naturally:

• Do the fundamental equations of fluid mechanics have solutions that describe the various vortical structures observed in experiments? Are these solutions dynamically stable, hence physically relevant? If not, under which process are they destabilised?

• How do vortices interact when they come close to one another? This question is linked with fundamental phenomena in fluid mechanics, such as the vortex merger in two dimensions and the vortex reconnection in three dimensions.

• How do vortices interact with an impermeable boundary, or with a solid advected by the fluid? This is also related to crucial practical questions, such as understanding the *ground effect* that allows to boost the lift and reduce the drag of airplanes flying near the ground.

With regards to these questions, it is worth pointing out the analogies between vortices and shear flows. For instance, it is well known that free surfaces in incompressible inviscid layered flows can be described as vortex sheets which allows to use commun technics for both theoretical [15, 77] and numerical [7] results. Boundary layers can be also viewed as vortex sheets along a solid boundary [54], and Kelvin-Helmoltz instabilities develop in the boundary layer near the Earth's magnetosphere [76]. In the vanishing viscosity limit, strong vorticity gradients near the core of the vortex prevent the use of direct numerical simulations. As in the case of boundary layers, asymptotic models are therefore necessary to understand and compute the dynamics of complex vortical structures. A famous example is the binormal flow equation, known also as VFE (vortex filament equation) and LIA (local induction approximation), on which numerics can be easily performed and is used as a standard model by physicists. It can notably describe the evolution of singular vortex filaments generated behind delta-wing aircrafts and non-circular jets.

All these analogies lead to mathematical problems which are similar, and can therefore benefit from common mathematical tools. This is the main rationale behind the SingFlows project.

Wave-structure interactions and marine energy

The sector of marine renewable energies has grown quickly over the recent years, with the promise of a green and significant energy source. Marine renewable energy encompasses many different technologies; some of them are already well implemented (off-shore wind turbines for instance), while others such as wave energy converters (WEC) are more complex and expensive and still require to be more thoroughly investigated. In this project, we will mainly focus our attention on the modelling, the mathematical analysis and the numerical simulation of WEC but many of the expected results will have an impact on other technologies as well. A primary concern is for instance the wear of WEC under extreme weather conditions, and this concern is shared by all kinds of offshore structures. This concrete question requires understanding the behaviour of floating structures in the context of water waves of high amplitude. This is a significant challenge from the mathematical and modelling point of view. On top of the water waves dynamics, which is already complex in itself, one needs to understand the evolution of the immersed part of the floating structure. This is of course a coupled dynamics: the water waves exert a force on the structure, which through its displacement retro-acts on the waves. Moreover, in the long term, one should describe a network of converters, therefore with more intricate coupling.

To the best of our knowledge, the current approach to simulate the dynamics of the converter relies on computationally expensive CFD simulations or softwares such as WAMIT, which are based on a linear theory, and limited to time harmonic forcing by the swell. This approach prevents from describing transient regimes and nonlinear effects, and does not allow to reproduce the dynamics of the contact line between the water surface and the solid structure. Going beyond this linear approach, obtaining a realistic model and encoding it are essential motivations for the SingFlows project. This generates difficulties that are similar to those encountered with anisotropic flows: the direct simulation of free surface Euler or Navier-Stokes equations is too heavy and the linear models are too simple. The challenge is to derive simpler sets of partial differential equations that are able to follow the evolution of the contact line and to take into account the other nonlinear effects. We shall meet here the same issue as for the first topic of SingFlows (flows with small aspect ratio): the choice between several possible models is difficult and must be based on modelling, and on mathematical and numerical considerations. Note that the justification of such derivations and the analysis of these new models raise very subtle and exciting questions in mathematical analysis: dynamics of the contact line, low regularity solutions, "turbulent closure" in reduced models, small viscosity effects, *etc.* Connections to other aspects of wave-structure interactions will also be established. The wave maker problem, for instance, will be particularly investigated.

1.2 Main objectives, originality and relevance in relation to the state of the art

The ambition of the SingFlows project is threefold:

1. *To develop new mathematical methods for fluid models at low regularity.*

Flows in boundary layers or near vortices become very singular at high Reynolds numbers. Standard mathematical methods for proving existence and uniqueness of low regularity solutions are often inapplicable in these situations, because they are based on Fourier analysis and do not take into account the geometrical nature of the singularity: sheets in the case of boundary layers, lines in the case of vortices. One must develop blow-up techniques that allow for a precise analysis of the singularity profile, and give information on its dynamical stability. In the study of boundary layers, this idea has been implemented through the use of fast/slow variables, which goes back to the pioneering work of Prandtl. Similar ideas were introduced to analyse vortices and filaments [14], but turning this into rigorous mathematics is a difficult open problem. Recently, the use of refined doubling-variable methods (semiclassical analysis, wavelets) has allowed for the analysis of inhomogeneneous problems in oceanography [17]. Also, self-similar variables were successfully used to study two-dimensional vortices and axisymmetric vortex rings [35, 31, 34]. One ambition of the project is to further develop such techniques, and notably to apply them in situations involving vortices interacting with rigid boundaries (a case for which no mathematical result is available so far). In more complicated situations where rigorous treatment of the primitive equations is out of reach, we shall analyze reduced models. Besides the binormal flow, we have in mind the system proposed in [78] and [55] to describe the interaction of nearly parallel vortex filaments. Studying singularities in such models is also a challenging objective, for which we will benefit from the innovative techniques set up in [10, 8, 34].

Contact lines for floating structures are another example of mathematical singularity. Contact lines also appear in the so-called shoreline problem, where the water depth vanishes. For the simplest reduced model (nonlinear shallow water), their analysis causes the same difficulties as vacuum in compressible Euler equations [48, 20]. But for more precise models including dispersion, the problem is harder and has been solved only in the 1D case [61]. For the full equations, another issue, considered in [25, 70] is the singularity of the fluid domain. These works are good starting points, but the contact line associated to a floating structure is far more complex: contrary to the shoreline, its evolution is not governed by a kinematic equation but by a much more singular PDE.

2. *To obtain effective reduced models for singular flows.*
In the context of anisotropic flows, we will use a two-step strategy. First, we will focus on boundary layer flows. Taking advantage of the recent mathematical impetus around the Prandtl equations [3, 39, 23], we will study the stability of other boundary layer models, coming from *interactive boundary layer theory* (IBL, see for instance [19]). These models are consistent with Prandtl, but behave better in the numerics [57]. Once the best models have been identified, we will in a second step incorporate them in the treatment of shallow water or pipe flows. The objective is to rely on these models to describe the interaction with the solid surface, so as to derive efficient closure relations, and in this way a good parametrisation of the friction. Our approach here is inspired by some very recent work of James, Lagrée, Le and Legrand on a modified shallow water model. See [47] for a preliminary version.
In the context of wave-structure interaction, we will follow a novel approach initiated by David Lannes in [59], where the interaction is described by a flow with partial pressure: the pressure exerted by the flow on the solid is understood as a Lagrange multiplier associated to the constraint exerted by the solid on the fluid surface. Note that this model is reminiscent of mixed compressible/incompressible models, in which other team members have expertise. Another aspect of our work will be to take into account several physical effects neglected in [59] such as viscosity or vorticity. Our hope is that these models will be both simple and accurate enough to simulate realistic configurations (for instance an array of WEC in a complex wave field) that are out of reach for direct CFD computations.

3. *To develop efficient numerical codes based on these reduced models.*
Reduced models can be extremely powerful, because part of the physical difficulties of the full problem have been handled analytically. It is however important to appreciate that even once their well-posedness has been established, their numerical implementation is usually not straightforward, often because regularising terms do not act on all directions. Great care must be taken in choosing the numerical scheme and building the resolution algorithm, in order to guarantee numerical stability. Our group includes numericists and physicists, who have over the years developed experience on this sort of numerically challenging computations. For instance, part of the team has already a strong experience in the simulation of water waves propagation, which will enable a rapid focus on the interaction with the floating structure. We expect our codes to go beyond theoretical interest, and reach industrial partners. At a more global level, we expect our project to spread to a larger community, especially to many young researchers and non-specialists.

2 Project organisation and means implemented

2.1 Scientific programme

We detail here the concrete tasks that we plan in order to reach the general objectives presented in section 1. We will provide a tentative schedule in the next paragraph.

Task 1. Beyond Prandtl : stability of interactive boundary layer models.

The boundary layer phenomenon is ubiquitous in flows at high Reynolds number (Re $\gg 1$). The classical Prandtl model, set up by Prandtl in 1904, was a breakthrough in our understanding of the boundary layer properties, but is not sufficient at least for two reasons. First, the unsteady Prandtl equation does not reproduce properly the growing modes of the Navier-Stokes equation, namely the Tollmien-Schlichting waves. Moreover, both in the unsteady and steady settings, the Prandtl solution blows up after separation. To overcome these issues, refined formal asymptotic expansions were proposed, that go under the name of triple deck models, or interactive boundary layer models (IBL). The general idea behind these models is to retain some $O(\mathrm{Re}^{-1/2})$ terms that are usually neglected in the Prandtl derivation. One gets in this way models that are consistent with the classical one, but which (at least in the steady regime) are more robust numerically. These reduced models have been extensively used in the 1980's and 1990's in aerodynamics, and have turned useful recently in other contexts [27]. Still, to the best of our knowledge, almost nothing is known from a theoretical point of view.

One task of the SingFlows project is to study mathematically the stability of these models. We would like to recover - at least for some of them - the observed fact that they are more stable than the Prandtl equation. Problems are to be considered gradually: from models with fixed displacement thickness to complete IBL models, from the analysis of simple linearizations to the local and then global well-posedness theory of the nonlinear equations. Such study was initiated recently in the unsteady regime in [21], but much remains to be done. We notably plan to focus on steady problems, for which most simulations were performed. We will first consider flows with no recirculation, and determine if the usual Crocco and Von Mises tranforms still allow for uniform bounds. We will then allow for recirculation. The point will be to check if the blow-up mechanism established in [23] for the classical Prandtl system is inhibited in the IBL context.

If the previous mathematical studies fail to exhibit good stability properties of the IBL models, they should help to understand the instability mechanism, and allow to add some appropriate stabilizing terms. Our objective is to identify at the end of this task some robust reduced boundary layer model. Such models can then be used in a more global system, as we will explain in the next paragraph.

Note that time permitting, Task 1 can be further extended. For instance, the methods that we will develop should apply to the so-called *RNSP model*, used recently in the modelling of thin pipe flows [58]. This model is a viscous version of the inviscid hydrostatic equations, which are known to be well-posed under concavity conditions on the velocity. It would be interesting to see if well-posedness is preserved with the addition of transverse diffusion.

Task 2. Improvement of friction modelling in anisotropic flows

A standard approach to 2D flows with small aspect ratio is as follows: i) one neglects the tangential diffusion in the evolution of the tangential velocity u, ii) one replaces the equation for the transverse velocity v by the so-called *hydrostatic approximation*, iii) one integrates the momentum equation for u over the water height h. This integrated equation involves the average velocity $h^{-1}\int_0^h u$, but is not closed. A major issue is to determine closure relations for the integrated convection term and integrated diffusion term, this last term being the tangential shear stress at the boundary. In the case of shallow water flows, the ideal Saint-Venant model assumes that the velocity is constant along the water height, and omits the tangential shear stress. On the contrary, a viscous Saint-Venant model can be obtained by imposing a parabolic (half-Poiseuille) profile for the velocity as the closure law [12]. Still, in most situations, these closure laws are far from accurate: they do not reflect the fact that the effect of the viscosity is localised in a boundary layer at the bottom, and that this effect is very sensitive to small bathymetry variations (typically a small bump). The usual solution is to add to the ideal Saint-Venant model an *ad hoc* friction term. This term is based on empirical considerations (Chezy's law, Manning's law), or on idealised Prandtl boundary layer theory [64]. A similar difficulty arises in pipe flows.

A main task of the SingFlows project will be to use interactive boundary layer models to improve the description of friction in anisotropic flows:

• We will start from a recent work of James, Lagrée, Le and Legrand, where an original enriched shallow water-system is proposed (*cf* [47] for preliminary results). It is based on a coupling between the ideal shallow water model and a Von-Karman equation, which is an integrated version of the Prandtl boundary layer equation. Closure laws are derived here by imposing polynomial profiles in the boundary layer. Although preliminary numerical tests are encouraging, much remains to be done on the relevance of this model (domain of hyperbolicity of the linearised model around constants, nonlinear well-posedness). Meanwhile, we will investigate its physical relevance by testing various choices of polynomial profiles and bathymetries.

• In a second step, we will try to replace the Prandtl equation by an interactive boundary layer model. The analysis performed in Task 1 will be crucial here to select the relevant system.

• Once an enriched shallow water model has been derived, the next step is to identify some possible law for friction in terms of the water height, bottom topography and average velocity. This goal can be pursued by analytical means or - more likely - through numerics. A breakthrough result would be to recover in this way some empirical formula from the literature (typically Chézy or Manning laws).

Task 3. Description of vortex interaction in slightly viscous fluids

We will provide qualitative information on the deformation and interaction of vortices in Navier-Stokes flows. We shall proceed gradually.

• *Single vortex filament.* Isolated vortices in $\mathbb{R}^2$ or filaments in $\mathbb{R}^3$ already have a non-trivial evolution under the Navier-Stokes dynamics. In two dimensions, the dissipation mechanism is well-understood [35, 32], and detailed stability results for Lamb-Oseen vortices are now available. We will therefore focus on the three-dimensional case, and will rely on the recent advances of Gallay and Sverak in [34], who have proved global well-posedness of the axisymmetric Navier-Stokes equation (without swirl) starting from a circular vortex filament. We believe that the techniques introduced in [34] are robust enough to handle more general configurations. Our intention is to start with perturbative settings, such as nearly rectilinear filaments or small perturbations of circular filaments [44]. Another objective, in the axisymmetric setting, is to show that the short-time asymptotic result of [34] remains valid over a fixed time interval $[0, T]$ in vanishing viscosity limit. This would provide the first rigorous justification of the binormal flow for viscous vortex rings.

• *Interaction of several vortex filaments.* When several point vortices are present in a planar viscous fluid, each vortex generates a non-uniform velocity field which destroys the circular symmetry of the other vortices and creates nonlinear interaction. The stability analysis conducted in [31] shows that the vortices are slightly deformed, so that the self-interaction exactly counterbalances the strain of the velocity field. A mid-term project is to extend this result to axisymmetric flows arising from circular vortex filaments, by combining techniques developed in [30, 31, 34]. This will in particular provide a rigorous justification of the leapfrogging phenomenon in slightly viscous fluids. Note that such a justification was recently obtained by Jerrard and Smets [53] for quantum fluids governed by the Gross-Pitaevskii equation, but the corresponding problem is open for both the Euler and the Navier-Stokes equations.

• *Interaction with material boundaries.* This interaction was extensively studied in physics, notably through experiments, but we are not aware of any mathematical result even in dimension two. In order to get a better understanding of possible interactions, we will first investigate the idealised but paradigmatic case of a point vortex moving in a half-plane. For an ideal fluid, the "method of image charges" gives an explicit formula for the velocity field, which predicts a uniform translation of the point vortex in the direction parallel to the boundary. However, viscous fluids are expected to behave in a completely different way, due to the interaction of the point vortex with the boundary layer. For instance, trailing vortices created by airplanes are observed to undergo a rebound on the ground, a phenomenon usually referred to as the "ground effect". Our aim is to shed some light on this effect, through a mathematical analysis involving the construction of an explicit correction, of boundary layer type, to the inviscid velocity profile. This is an instance where collaboration between experts in boundary layer theory and vortex dynamics will be fruitful.

• *Interaction with shear.* The interaction of a vortex with a shear is extremely intricate. It can notably result in a weakening of the vortex, like in the case of the Earth atmosphere, in which tropical cyclones are significantly weaker in the presence of an ambient atmospheric shear. We want to develop a numerical scheme to investigate the stability of a vortex line subject to a transverse shear. Our goal is to obtain a bifurcation diagram for the vortex strength in terms of the temperature and the shear.

Task 4. Singularity formation on vortex structures *via* reduced models

Proving the stability of singular structures such as vortex filaments in inviscid 3D fluids seems for the time being out of reach without stringent assumptions [51]. The slightly viscous case is the core of Task 3. To improve on the description of vortex dynamics, we shall rather consider here asymptotic models or restrict ourselves to special classes of solutions: self-interaction of one filament through the binormal flow approximation and interaction of several vortices (points or filaments).

• *Vortex filaments with corners.* In a recent series of papers (see [10] and references therein), Banica and Vega have provided an accurate description of the evolution of one single corner in a vortex filament evolving according to the binormal flow equation. Now that this mechanism is well-understood, we wish to tackle the case of filaments that are initially concentrated on a polygon. A chaotic dynamics, involving rotation of the axis and resurgence of symmetry, is observed experimentally (for instance in non-circular wake jets) and in numerical calculations [43, 52]. It was proved in [24] that the filament should be a skew polygon at rational times. Inspired by the paper [46], we plan to show that the evolution of each corner in the initial polygon captures some characteristic features of turbulence theory, such as multifractality *via* the Frisch-Parisi conjecture. We also aim at understanding what the evolution should be for irrational times, and eventually prove that the constructed curve evolution is a weak binormal flow solution in the sense of [52].

• *Interaction between several vortices.* Significant progress was made recently [9, 8] about the classical model for almost parallel vortex filaments due to Zakharov [78] and Klein *et al* [55]. Nevertheless, this model has limitations. It would be of interest to take into account the self-induced effect of each filament during the reconnection process. Another important topic in spray models is the modified point vortex system derived in [41], in which the vortex centers are accelerated by a Kutta-Jukowski lift force. For instance we aim at proving that, generically, the vortices do not collide in finite time (see [66, Chap. 4.2] for the classical point vortex system).

Task 5. Wave-structure interactions

One of the main novelties when one adds a floating structure at the surface of a fluid - otherwise governed by the water waves equations - is that a new free boundary problem arises, namely, the evolution of the contact line between the water, the solid and the air. We shall investigate first the case where the solid is in forced motion – the more complex case of freely floating objects will be investigated afterwards.

• *Theoretical analysis of the contact line.* The position $\underline{x}(t)$ of the contact line is defined implicitly by the relation $h(t, \underline{x}(t)) = h_{\mathrm{w}}(t, \underline{x}(t))$, where h is the water height and h_{w} is the distance from the wetted surface of the solid to the bottom - in the case of forced motion, this function is known. The main difference with the shoreline problem [48, 20, 61] is that the boundary condition at the contact line is *not* kinematic anymore, so that for instance Lagrangian methods do not seem applicable. In addition to the contact line itself, it is necessary to address the well-posedness of the fluid equations in the inner (under the solid) and outer regions. We will rely on the coupled compressible-incompressible formulation introduced in [59], see paragraph 1.2. This is an entirely new problem, involving a new kind of free boundary (more singular for instance than the one related to the stability of shocks).

We shall consider it by addressing situations of increasing complexity: $1D$ before $2D$, nondispersive models like NSW before weakly nonlinear models like Boussinesq, and finally the full water waves equations. In the latter configuration, it will be necessary to analyse the potential equation in the fluid domain which has a corner (or an edge) at the contact line; good starting points for the analysis will be [70, 25]. Our study should also benefit from recent advances in the study of congested flows - see Task 6 - or from recent progresses in the analysis of mixed initial-boundary value problems for dispersive equations [5, 6]. In return, the results that we will obtain will hopefully apply to a larger class of models.

• *Full fluid-structure interaction.* After a good understanding of the contact line, we plan to compute the retroaction of the fluid on the floating structure. This means that we need to include in Newton's law the hydrodynamic force exerted on the solid. This amounts to computing the pressure at the solid surface. A key step to avoid numerical instabilities and to allow well-posedness is to exhibit the added-mass effect in this pressure contribution [16, 41]. A main point will be to handle this added mass effect in models of increasing complexity.

• *Viscous regularization.* Besides the treatment of the inviscid model above, which is very challenging, another natural approach is the analysis of a viscous version. There exists indeed a now classical theory for the free boundary Navier-Stokes equations, *cf* [75] and references therein. We think that we should be able to generalize these techniques in the presence of a second free boundary (the contact line) based on our expertise in fluid-structure interactions for totally immersed solids [28, 41, 67] and elastic membranes [42]. A further step forward would be the understanding of the vanishing viscosity limit: article [68] could be helpful in that. Furthermore, the study of associated boundary layers and friction effects could draw a connection to Tasks 1 and 2.

• *Numerical study of wave-structure interactions.* The first difficulty is to have a good description of the evolution of the contact line. As a Lagrangian description is ill-suited for this, we shall rather treat it by means of an embedded boundary method for which the contact line is implicitly represented on an unfitted mesh by means of a level set method for instance. The key element here (e.g. [71]) is to construct a conservative coupling between the different flow regions. We shall also devise sufficiently accurate and stable ODE techniques to evolve in time the parametrization of the contact line, consistent with the nontrivial dynamics studied theretically in the previous point.

For the fluid-structure interaction itself, there are mainly three aspects. The first involves the characterization of the non-linear wave hydrodynamics on non-moving structures. For this, we will rely on existing efficient schemes designed by some team members. They are based on adaptive unstructured grids for fully nonlinear and weakly dispersive wave models [60, 29].

The second aspect is the coupling between these outer wave models and the inner flow equations delimited by the contact line. The equations are incompressible in this inner region, and an elliptic problem must be solved for the pressure on the body. A rather classical variational approximation of this problem will be considered with Dirichlet boundary conditions obtained from a continuous finite element extrapolation within the cells containing the contact line [63].

A third aspect arises for freely floating objects. In this case, several developments are possible. These involve first of all the choice of the time stepping strategy (implicit, partitioned or explicit), the use of explicit time stepping techniques explicitly accounting for added mass effects in the discrete ODEs for the body. Initial studies will be performed on two-dimensional configurations with heave motion, before passing to three-dimensional geometries, and to horizontal movements. This task overlaps with some of the activities under way in the European project MIDWEST aiming at proposing a hierarchy of modelling tools for floating bodies. Our interaction with the consortium of MIDWEST, and with some SMEs in the domain of wave energy (e.g. TECNALIA) will provide us with guidelines to study configurations relevant in real applications (see *Technological transfer* in §2.4). Discussions with the team of Roberto Camassa (Chapell Hill) are also underway in order to set up experiments in a wave tank, that would serve as benchmarks for the computations.

Task 6. Connection to related models

We want here to draw parallels to related problems of high interest. These problems share features with those evoked in Task 5, but rely on different mathematical tools (control theory, compactness methods). We expect that conducting Tasks 5 and 6 in an interactive manner will be a source of innovative ideas.

• *The wave-maker problem: wave generation.* We want to address the question of the generation of water waves in a numerical or experimental wave tank. This corresponds to the mathematical problem of the controllability of the water-wave equations. Recently, a local exact controllability result was obtained for free surface Euler equations with surface tension [2]. It proves that one can generate arbitrary small amplitude periodic water waves by blowing on a localized portion of the free surface of a liquid. We plan to extend this result to experimental configurations where water waves are produced either by the immersion of a solid body (called plunger), or by the oscillation of a solid portion of the boundary. In order to treat these situations, we will combine the recent estimates for bounded domains derived in [25] and the boundary observability inequalities due to Alazard [1].

• *The wave-maker problem: artificial boundary conditions and wave absorption.* Laboratory experiments of water waves face the difficulty of wave reflection against the boundaries of the basin. A similar problem appears in simulations, since for obvious computational reasons one has to work in a bounded domain. Our goal here is to bring some mathematical insight to this issue. We will consider the two classical approaches to the numerical treatment of unbounded domains. The first one consists in truncating the domain by the set up of an artificial boundary. This is a good strategy if one can find some special non-reflecting boundary conditions which make the artificial boundary invisible to outgoing waves. Such artificial boundary conditions have been studied recently in [50, 49] in the case of linear water waves, and we wish to extend their analysis to the nonlinear setting: we will rely on the nonlinear microlocal analysis of the water-wave equations performed in [2]. A second method consists in damping outgoing waves in an absorbing zone surrounding the artificial boundary. This is a numerical analogue to natural mechanisms, such as energy dissipation created by a beach with a mild slope. Mathematically, this corresponds to the stabilization of the water-wave equations. Our goal here is to work on the models and methods developed by the oceanographers from Ecole Centrale de Nantes to simulate numerically experimental absorbers. In particular we want to study the wave-structure problem where water waves interact with a porous medium.

• *Connection to congested flow models.* The mixed compressible/incompressible model from [59] is reminiscent of the so-called *hard models* used in the simulation of congested flows [69, 73] . These models come from a hydrodynamical modelling of very diverse phenomena, like traffic jams, blood circulation, crowd motion, etc. (see for instance [26, 69, 72, 74]). They oppose to the *soft models*, which are typically compressible equations in which congestion is described by singular pressure laws. The transition from soft to hard models is made formally by adding a small parameter in front of such singular pressure term and sending this parameter to zero [13]. The analogy with the models introduced in [59] can prove very useful for the analysis of situations where the wetted surface of the solid changes connexity. Conversely, we expect that improving our understanding of the dynamics of floating structures will benefit to more general hard models with time and space dependent packing constraints. Time permitting, we shall also try to include the effect of viscosity, that may generate new phenomenology such as memory effects [72, 73, 62].

2.2 Actions

Task implementation

The tasks described in the previous paragraph will correspond to different periods of the project, and different duration, depending on their exploratory nature.

Task 1 (stability analysis of IBL models) should be realised over the first two years of the project. We believe that this short term objective is reasonable: strong progress was made recently in the analysis of classical boundary layer problems, and our task force has already much expertise and interaction on these questions.

Task 2 (improving the modelling of friction) requires that the accomplishment of Task 1 is advanced. Therefore, it will start in year 2 of the project, and end in year 4. More precisely, the analysis and simulation of the enriched shallow water model of [47] will be considered in year 2, the derivation of new models including IBL will be considered in year 3, while extensive numerical computations to infer friction laws would be the objective of the last year.

Task 3 (vortex dynamics in viscous fluids) is decomposed in several stages. In the first two years of the project, we plan to understand the dynamics of a single axisymmetric vortex filament in the vanishing viscosity limit (t fixed, $\nu \to 0$), and to justify the binormal flow in this context. In a second stage, our goal is to use that result to describe interactions of circular filaments, leading to the leapfrogging phenomenon. In parallel, we want to investigate the evolution of more general structures in perturbative settings, including nearly rectilinear or nearly circular filaments. As for the interaction of a vortex with a wall, a first step - to be initiated at the kick-off conference - is to identify a good functional setting and to build an appropriate model for the boundary layer. Understanding the motion of the point vortex in a slightly viscous regime will be done in a second stage, taking advantage of the fact that various team members are experts in vortex stability and/or boundary layer theory. Another interesting question is the long time behaviour of the solutions in this context, which could be studied following earlier results in [33, 45].

Task 4 (vortex dynamics in reduced models) includes two mid-terms objectives. We hope to enlighten within two years the complex algebraic structure which governs the evolution in time of a polygonal curve through the binormal flow, and which generates the fractal dynamics detected in the numerics. In parallel, we shall study the occurence of collisions in the extended point vortex system derived in [41], and hopefully prove that no collisions occur for almost all initial data. We emphasize here that several team members are experts in collision problems, see [9, 56, 38]. Deriving and studying more accurate models for the interactions of almost parallel filaments is an ambitious problem, which we plan to address in a second stage, during years 3 and 4 of the project.

Task 5 (dynamics of floating structures) will be considered gradually. During the first two years of the project, we expect to tackle the contact line problem for (NSW) and (SGN), in horizontal dimension $d = 1$. We shall then investigate the case $d = 2$ and/or the full water waves equations. The numerical study will follow the same rythm: in the case $d = 2$ the case of purely vertical motion (heave) will be considered first. The treatment of the full viscous model should be tractable within the first half of the project, in parallel to the analysis of the contact line. In the inviscid case, understanding the solid dynamics will of course depend on the progress realised on the contact line problem. Nevertheless, we will be able to work independently on this topic by considering solids with vertical walls, since the dynamics of the contact line is trivial in this case.

Our analysis of the wave-maker problem, which is a central objective of *Task 6* will first focus on the design of adapted artificial boundary conditions. Accurate wave-maker models will be considered afterwards. The study of congested phenomena will be more transversal, and carried during the whole duration of the project. Regular meetings will be organised between the members involved in tasks 5 and 6 to favour the transfer of methodologies between the tasks.

Dissemination of the results

Obviously, SingFlows is not conceived as a self-sufficient project. We will be careful to have strong connections within the scientific community, but also with potential industrial partners, and with non-specialists.

Scientific communication. To ensure a link with the international experts on the same topics, we plan to invite foreign researchers throughout the duration of the project: we have in mind that each partner should invite 1 to 2 foreign researchers per year, for two-weeks stays. We also plan to propose a mini-symposium at the international conference *Equadiff 2019*. Furthermore, we will organize two conferences and a summer school, reflecting the three main themes of the project (anisotropic flows, vortex flows, wave-structure interaction). These three events will mix research talks and lectures to make them attractive to PhD students and postdocs. Moreover, we plan to reach an audience of non-specialists, either through publications in popular science journals - some of the team members have already contributed to magazines such as *Pour la Science*, or *La Recherche* - or through popular science conferences (like the Maths Club of University Paris Diderot or the Semaine des Mathématiques for high school students).

Technological transfer. One ambition of the SingFlows project is to be of an applied nature, and our activities around the theme of wave-structure interactions are very promising in this respect. These activities benefit from the interaction with european specialists in the domain of wave energy conversion, *via* a collaboration with the consortium of the MIDWEST OceanERANET project (`https://project.inria.fr/midwest/`), coordinated by M. Ricchiuto. Moreover, D. Lannes and M. Ricchiuto have initiated another collaboration with the technology transfer agency Tecnalia, in order to identify the problems that are most relevant for practical applications. All these interactions will allow us to compare the models we will develop in the panorama of existing industrial and commercial tools, and to benchmark them against these products. A first round table discussion on these issues will take place during the international workshop HYWEC, co-organised with BCAM, and with the MIDWEST consortium (`http://www.bcamath.org/en/workshops/hywec2017`).

3 Impacts and benefits of the project

3.1 Theory

Obviously, most of the objectives of the SingFlows project aim at a theoretical impact, notably towards partial differential equations. We think that the mathematical tools that we will develop for localized structures, such as vortex filaments or boundary layers, will overall benefit to mathematical fluid dynamics: for many years, the dominant trend to tackle fluid mechanics problems has relied on Fourier analysis, and there is a need for more spatially inhomogeneous methods. In the same spirit, the improvement that the project will bring to the treatment of wave-structure interaction will benefit to the general theory of mixed problems for nonlinear dispersive equations.

Specifically, we expect the following mathematical results in the mid/long term:

• Accomplishing Task 1, we will be able to give a rigorous justification to the use of interactive boundary layer and triple deck theories. In this way, we will bridge a gap between the mathematical community and the mechanics/engineering community where these theories have been used successfully - without much justification.

• Accomplishing Task 2, we will obtain an improved modelling of friction in shallow water or pipe flows, *starting from the Navier-Stokes equations.* This will provide a significant improvement compared to most derivations (see [40, 65]), where the friction terms are given *a priori*, mostly through empirical laws. A very stimulating perspective, probably more in the long term, is to be able to derive a simple dimensional friction law, involving the water height and the mean velocity. This law will then be included in the usual Saint-Venant system. A breakthrough would be to recover in this way the usual Chézy's or Manning's empirical laws.

• As regards the dynamics of vortices, we expect significant progress in at least two directions: towards a rigorous justification of the binormal flow, and towards a mathematical description of the interactions between vortices and material walls. The first one is a fundamental open problem in fluid mechanics, and the second one is closely related to very important practical questions, such as understanding the origin of the ground effect in aerodynamics. Solving these problems will require a better understanding of the dynamics of the Navier-Stokes equations in a non-perturbative regime, and we certainly expect that the techniques developed here will be useful to attack other problems in the future. On the other hand, deriving more realistic models for the interaction of two-dimensional vortices or three-dimensional filaments is a modelling issue of the utmost importance, which may shed a new light on extremely difficult open questions in vortex dynamics: the merger of planar vortices, or the reconnection of vortex tubes.

• Concerning the mathematical analysis of wave-structure model, the long term hope is a full mathematical understanding of the system introduced in [59]. In our efforts towards this goal, we will have to solve open problems of independent interest on compressible-incompressible models - that arise in other situations involving congested flows such as granular media, traffic jams, social hydrodynamics - and on mixed initial-boundary value problems for dispersive perturbations of hyperbolic systems - very important for many applications in hydrodynamics.

3.2 Applications

The SingFlows project is centered on topics of practical importance. It offers an integrated approach that mixes qualitative mathematical analysis, numerical schemes and computations. For these reasons, applications of the theoretical results are highly expected:

• Indeed, improving the calculation of wall shear stress in anisotropic flows should be helpful in various areas, notably in the study of erosion processes, drag computation, or in the study of the blood circulatory system. For instance, in arteries, elevated wall shear-stress due to stenoses may initiate the mechanism of thrombo-embolism. The gain of computational time by using reduced models is significant and may be useful if rapid diagnosis is needed [58, 18]. A tractable but accurate parametrisation of friction is also crucial to many domains, for instance in atmospheric sciences to describe vortex streets in the lee of large isolated islands. We are optimistic that the codes to be developed during Task 2 will be a remarkable improvement to the existing simulations.

• The interaction of an isolated vortex with an ambient shear will find applications in the context of tropical cyclones, which are huge atmospheric vortices, driven by moist convection and impeded by an ambient atmospheric shear. The effect of the shear on a tropical cyclone strength is very poorly understood. We expect our codes to improve the prediction of this strength through the computation of a bifurcation diagram in terms of the control parameters (temperature, shear strength).

• Our project should also lead to significant advances in the simulation of floating structures. Firstly, our approach allows one to find the pressure exerted on the solid by solving an elliptic equation in d dimensional bounded domain (d is the horizontal dimension) while the standard approaches require the resolution of the $d+1$ dimensional potential equation in an unbounded domain (the direct CFD approaches being even more computationally demanding). The gain in computational time should allow us to provide numerical computations for complex configurations such as arrays of wave energy convertors in complex and large amplitude wave fields. Moreover, the equations used by engineers for the motion of the floating structure are based on an oversimplified analysis, and we will propose significant corrections. This is a great gain compared to the usual three-dimensional modellings of the same problem. This is obviously very appealing from a numerical point of view. We clearly expect that, by the end of the SingFlows project, the design of a numerical code based on this system will be achieved. For these aspects, we will be in direct contacts with some of the most significant actors in wave energy engineering through the network Midwest led by M. Ricchiuto and our contacts with the technological agency Tecnalia (see paragraph 2.2). We are confident that in the long run our work will have an impact for engineers.

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